Majorization and the Lorenz Order with Applications in Applied Mathematics and Beyond

Abstract

Majorization and the Lorenz Order with Applications in Applied Mathematics and Economics (Statistics for Social and Behavioral Sciences) by Alan G. Robinson

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Majorization is a partial order on vectors that measures the extent to which one vector is "more spread out" than another. The Lorenz order is a related partial order that measures the extent to which one vector is "more concentrated" than another. These orders have a wide range of applications in applied mathematics, including statistics, economics, and finance.

In this article, we will provide a gentle to majorization and the Lorenz order. We will discuss their definitions, properties, and applications. We will also provide some examples to illustrate how these orders can be used to solve real-world problems.

Definitions

Majorization

Let \(x\) and \(y\) be two vectors of length \(n\). We say that \(x\) is majorized by \(y\),denoted \(x \prec y\),if the following conditions hold:

$\sum_{i()}^{i} x_i \geq \sum_{i()}^{i} y_i$, for all $1 \leq i \leq n-1$ (monotonicity) $\sum_i x_i = \sum_i y_i$ (equal sums)

In other words, \(x\) is majorized by \(y\) if the cumulative sums of their components are non-decreasing and their sums are equal.

Lorenz Order

Let \(x\) and \(y\) be two vectors of length \(n\). We say that \(x\) is Lorenz ordered by \(y\),denoted \(x \prec_{L}y\),if the following conditions hold:

$\sum_{i()}^{i} x_i / \sum_i x_i \geq \sum_{i()}^{i} y_i / \sum_i y_i$, for all $1 \leq i \leq n-1$ (monotonicity) $\sum_i x_i = \sum_i y_i$ (equal sums)

In other words, \(x\) is Lorenz ordered by \(y\) if the cumulative proportions of their components are non-decreasing and their sums are equal.

Properties

Majorization and the Lorenz order have a number of interesting properties. Some of the most important properties include:

- **Transitivity:** If \(x \prec y\) and \(y \prec z\),then \(x \prec z\).
- **Reflexivity:** \(x \prec x\).
- **Antisymmetry:** If \(x \prec y\) and \(y \prec x\),then \(x = y\).

- **Linearity:** If $x \prec y$ and $a,b \geq 0$,then $ax + by \prec ay + bx$.
- **Monotonicity:** If $x \prec y$,then $x + z \prec y + z$.
- **Convexity:** If $x \prec y$ and $z \prec w$,then $(x + z)/2 \prec (y + w)/2$.

These properties make majorization and the Lorenz order very useful for a variety of applications.

Applications

Majorization and the Lorenz order have a wide range of applications in applied mathematics. Some of the most common applications include:

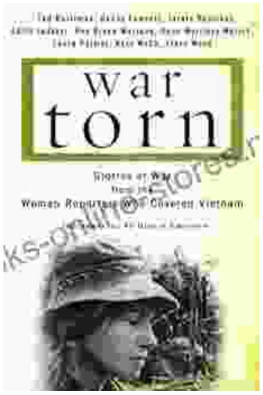
- **Statistics:** Majorization and the Lorenz order can be used to compare the distributions of two or more random variables. For example, they can be used to test whether two distributions are equal, or to compare the dispersion of two distributions.
- **Economics:** Majorization and the Lorenz order can be used to measure economic inequality. For example, they can be used to compare the income distributions of two or more countries, or to track the changes in income inequality over time.
- **Finance:** Majorization and the Lorenz order can be used to measure the risk of a portfolio. For example, they can be used to compare the risk of two or more portfolios, or to track the changes in risk of a portfolio over time.

Majorization and the Lorenz order are powerful tools that can be used to solve a variety of problems in applied mathematics. By understanding the definitions, properties, and applications of these orders, you can use them to gain insights into a wide range of real-world phenomena.

Examples

Here are a few examples to illustrate how majorization and the Lorenz order can be used to solve real-world problems.

1. **Example 1:** Suppose we have two distributions of income, \(X\) and \(Y\). We want to test whether the two distributions are equal. We can use majorization to test this hypothesis. If \(X \prec Y\),then we can conclude that \(X\) is more concentrated than \(Y\). This would suggest that the distribution of income is more unequal in \(X\) than in \(Y\).
2. **Example 2:** Suppose we have two portfolios, \(A\) and \(B\). We want to compare the risk of the two portfolios. We can use the Lorenz order to compare the risk of the two portfolios. If \(A \prec_{L}B\),then we can conclude that \(A\) is more risky than \(B\). This would suggest that \(A\) is more likely to experience large losses than \(B\).
3. **Example 3:** Suppose we have a time series



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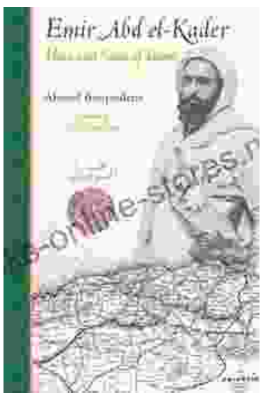

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